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MATHEMATICAL MODELLING OF BAGNOLD EFFECT AND SEPARATION BEHAVIOR OF A COAL-WASHING SPIRAL USING A MECHANISTIC APPROACH

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ABSTRACT

Among the gravity separators, the spiral concentrator is considered to be one of the most efficient and simple unit operations. Complex mechanisms, including the combined effects of different forces, differential particle settling rates, interstitial trickling and possibly hindered-settling affect the stratification of particles. "Bagnold effect" arises at relatively high pulp densities of the feed. This effect is attributed to the velocity distribution along the depth of the flowing film, which gives rise to a distribution of shear rates along the depth of flow. An improved mathematical model incorporating Bagnold effect is proposed to characterize the separation behavior of a coal-washing spiral. Sensitivity studies of the operating parameters on the segregation behavior of particles across the radial width of the trough during their motion along the spiral have been carried out. It is believed that the model reasonably mimics the segregation behavior of a particle during its motion along the spiral with a reasonable degree of realism.

Keywords : Spiral, Mathematical modelling, Coal washing, Mechanistic approach.

INTRODUCTION

Among the gravity separators, the spiral concentrator is considered to be one of the most efficient and simple unit process. Because of its relative simplicity and high efficiency compared to other gravity separators, it has been widely used under a variety of circuit configurations for processing of minerals and coal. Since its introduction by Humphreys in the 1940's, spirals have proved to be a cost effective and efficient means of concentrating a variety of ores. Their success can be attributed to the fact that they are perceived as environment friendly, rugged, compact, and cost effective[1].

In spiral operation, a high level of turbulence influences the Bagnold force component and distorts the steady-state flow conditions and spatial uniformity of the pulp. Any transients in the flow field affects the Bagnold force experienced by the particle. The Bagnold forces, in principle, acting on particles located at different depths, having different sizes and densities will have different magnitude for each particle in the pulp. Here, a mathematical model based on the previous approach[1-2] has been developed and modified to incorporate the Bangnold effect[3,4], to predict equilibrium distribution of prediction of relative specific gravity of the particle, particle size as a function of design parameters, mean flow depth and operating conditions of a coal washing spiral. Sensitivity studies of the process/operating parameters on the segregation behavior of particles across the radial width of the trough during their motion along the spiral have been carried out.

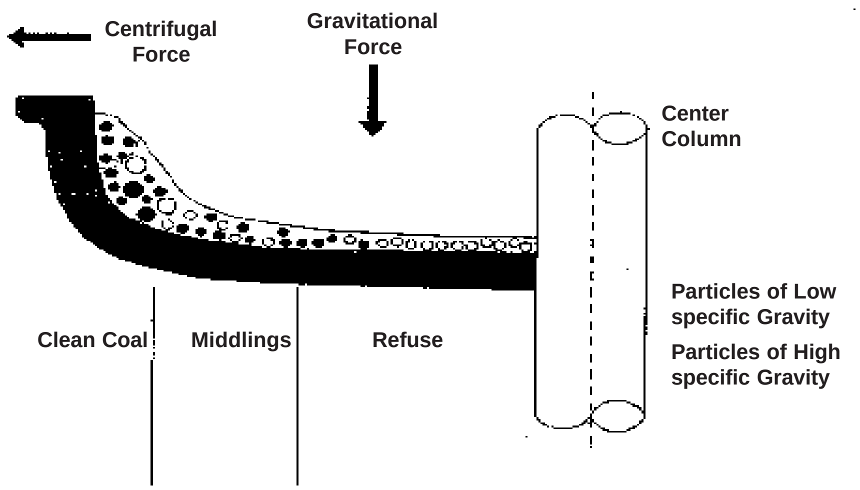

Fig. 1 : A sectional view of the a spiral trough flow.

MATHEMATICAL PROCESS MODELING

The modeling framework of the coal-washing spiral is constitutes of parametric representation of spiral geometry, treatment of fluid flow on spiral and particle force balance analysis, which are coupled through design, and operating parameters.

Parametric Representation of Spiral Geometry

The performance of the coal-processing spiral is a critical function of its design parameters, which include diameter, height, number of turns, pitch and slope as well as the shape of the trough and its dimension.

The parametric representation of a helix in the Cartesian coordinate system is given by the following equations [5]

$$x = r\cos[\eta]; \quad 0 \le \eta \le N\pi \tag{1}$$

$$y = r\sin[\eta] \tag{2}$$

$$z = \frac{U}{2\pi}\eta; \quad 0 \le z \le H \tag{3}$$

where, N is twice the number of turns, H is spiral height and r is the radial distance from the central line. η is a parameter used for parametric representation of the co-ordinates.

Treatment of Fluid Flow on Spiral

The spirals exhibit one of the most complex flow regimes among gravity separators used in mineral and coal processing operations. Spiral concentrator flows possess a free-surface, have shallow depths of <1 cm typically, and display laminar to increasingly turbulent flow behavior radially outwards with velocities reaching about 3-4 m/s[6,7].

Power Law Formalism for Flow Analysis

A more practical approach to flow modeling in a spiral relies on velocity profiles as classified by assuming appropriate flow regimes[8,9]. The action of centrifugal force on water as it flows down the spiral channel has two important consequences. First the water level at the outer concave wall of the trough exceeds that at the inner convex surface. Secondly, a transverse secondary circulation is generated in the form of a helical spiral and its forward movement is similar to a corkscrew motion. The angles that the inward and the outward-bound flows make with the mean axial flow are functions of depth and radial position. The reported measurements[10] for these angles have only limited accuracy.

The final governing equation for calculation of mean flow depth, h_m, is derived[1,2] in the form of a bi-quadratic equation involving h_m in the following form :

$$h_m^4 - 2(c_x + c_y)h_m^3 + (c_x + c_y)^2 h_m^2 - \frac{Q(2d_p^{1/6})}{3.3\pi^{1.5}(\frac{U}{c_x})^{1/2}} h_m - \frac{Q(2d_p^{1/6})}{3.3\pi^{1.5}(\frac{U}{c_x})^{1/2}} \pi r_m = 0 \tag{4}$$

where, r_m is the mean radial position from the central line i.e, $r_m = (r_o + r_i)/2$. The coefficients of the above equation contain design and operating parameters.

Bagnold Effect

Bagnold effect is a consequence of occurance of Bagnold force during the motion of the particle in the helical path of the spiral. It is attributed to the velocity distribution along the depth of the flowing film, which gives rise to a distribution of shear rates along the depth of flow. As a result, the Bagnold forces, acting on particles located at different depths, having different sizes and densities, will be different in magnitude for each particle in the pulp[3]. This force acts to disperse particles, to separate them from each other, its magnitude is directly proportional to the shear rate and the square of the particle diameter. Its theoretical magnitude is given by :

$$F_{Bag} = 0.04\sigma(C_l d_p)^2 (\frac{du}{dy}) \tag{5}$$

where, F_{Bag} is the Bagnold dispersive force exerted on a particle, u, is the velocity, σ, the particle density; du/dy, the shear stress applied, d_p, the particle diameter and C_l is the linear concentration of particles, which is given by :

$$C_l = \frac{1}{(\frac{C_{max}}{C})^{1/3} - 1} \tag{6}$$

where, C is the volume concentration of solids and C_{max} is the maximum possible concentration. Calculation of Bagnold force necessitates knowledge of applied shear stress.

Mechanistic Force Balance Analysis

The forces acting on a particle during its motion in the spiral separator are critical to decide the dynamics of the particle and segregation behavior of the particles spectrum. It is not easy to identify and quantify most of these forces precisely. In general, only rough estimates of the five principal forces involved, namely, gravity, centrifugal, drag, lift and friction forces can be made[1,2,11]. In addition, the so-called "Bagnold effect" arises at high pulp densities of the feed.

The static equilibrium analysis provides an understanding of segregation of particles according to their density and size during their descent along the helical path of the spiral. The longitudinal component of all principal forces F_L, (excluding Bagnold effect) acting on a particle in steady motion is given as[2].

$$F_L = F_g \sin[\theta]\sin[\alpha] - F_c \cos[\theta]\sin[\alpha] + F_d \cos[\delta] - F_N \tan[\phi] = 0 \tag{7}$$

where, F_g is the gravity force, F_c is the centrifugal force, F_d is the drag force and F_N is the normal component of all forces. The normal component of the force is given as :

$$F_N = F_g \cos[\theta] + F_c \sin[\theta] - F_1 \tag{8}$$

where, F_l is the lift force acting on the particle. The transverse component of forces acting on an immobile particle is :

$$F_T = F_c \cos[\theta]\cos[\alpha] + F_d \sin[\delta] - F_g \sin[\theta]\cos[\alpha] = 0 \tag{9}$$

The relative specific gravity has been calculated as a function of equilibrium radial position which incorporates the design parameters such as longitudinal tangential slope S, slope angle α, local slope angle θ and mean deviation angle δ.

$$\frac{(\sigma-\rho)d_p g}{6\rho} - 0.04\frac{\sigma}{\rho}(\psi d_p)^2\frac{du}{dy}$$
$$= \frac{6Uh_m \cos(\tan^{-1}(\frac{c_y}{r_0-r_i}\tan\arcsin[\frac{r-r_i}{r_0-r_i}])}{4d_p g\tan\phi\sqrt{(U)^2+(2\pi r)^2}}[k_1\tan\phi + \cos(\tan^{-1}(11\frac{h_m}{r})) +$$
$$(\sin(\tan^{-1}((11\frac{h_m}{r})))(\frac{U}{2\pi r}) + \tan\phi\sec(\tan^{-1}(\frac{U}{2\pi r}))\sin(\tan^{-1}(11\frac{h_m}{r}))(\frac{c_y}{r_0-r_i}\tan\arcsin[\frac{r-r_i}{r_0-r_i}])] \tag{10}$$

The final expression for particle size is derived as follows :

$$d_p^{\,2} + \frac{(\sigma-\rho)gd_p}{6\rho A} = \frac{1}{A}\,\frac{6Uh_m \cos(\tan^{-1}(\frac{c_y}{r_0-r_i}\tan\arcsin[\frac{r-r_i}{r_0-r_i}]))}{4d_p g\tan\phi\sqrt{(U)^2+(2\pi r)^2}}[k_1\tan\phi+\cos(\tan^{-1}(1 \ldots)$$

$$(\sin(\tan^{-1}((11\frac{h_m}{r})))(\frac{U}{2\pi r})+\tan\phi\sec(\tan^{-1}(\frac{U}{2\pi r}))\sin(\tan^{-1}(11\frac{h_m}{r}))(\frac{c_y}{r_0-r_i}\tan\arcsin[\frac{r-\ldots}{r_0-\ldots} \ldots)]] \quad (11)$$

where, $A = -(0.04\frac{\sigma}{\rho}\psi^2\frac{du}{dy})$

The relative specific gravity is defined as the ratio of difference between particle and water densities and density of water i.e. $(\sigma-\rho)/\rho$.

NUMERICAL IMPLEMENTATION

The model is implemented in a C++ code to compute geometric parameters, mean flow depth, distribution of relative specific gravity and particle size as a function of radial equilibrium position. The design data of a typical coal-washing spiral, located in the pilot plant of the laboratory (NML), has been used in this simulation. The data are given in Table 1. The volumetric feed rate (Q) is taken in the range of 0.3 to 0.6m^3/hr. Patherdih (Jharkhand) coal fines are used for the washability studies.

Table 1 : Design data of the coal-washing spiral (NML)

(i)	Height (H) = 2.5 m	(ii)	Pitch (U) = 0.425 m
(iii)	Slope (S) = (tan α) = 0.17	(iv)	Outer radius (ro) = 0.48 m
(v)	Inner radius (r_i) = 0.08 m	(vi)	Trough slope (tan α) = 0.2
(vii)	Max. depth (c_y) = 0.15 m	(vii)	Radial width (c_x) = 0.4m

RESULTS AND DISCUSSION

Figs. (2-3) show the distribution of relative specific gravity as a function of equilibrium radial position, for different particle sizes, such as 1 mm, 1.25 mm, 1.5 mm, 1.75 mm and 2 mm respectively and corresponding mean flow depth are 5mm and 4 mm.

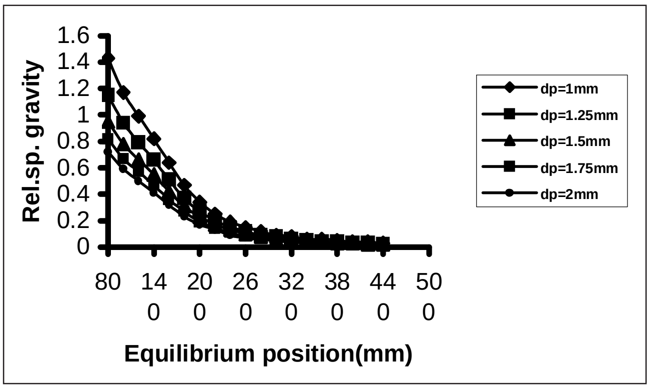

Fig. 2 : Effect of particle size variation on the radial distribution of relative specific gravity (feed mean flow depth = 5 mm).

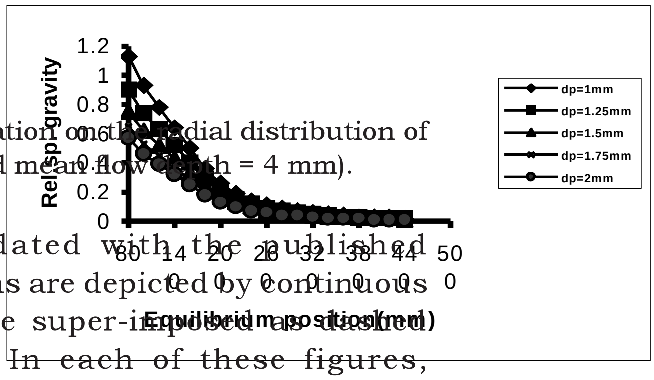

Fig. 3 : Effect of particle size variation on the radial distribution of relative specific gravity (feed mean flow depth = 4 mm).

The model has been validated with the published literature[1,2]. Model predictions are depicted by continuous line and literature values are super-imposed as dashed lines as shown in Fig. 4. In each of these figures, distribution of relative specific gravity as a function of equilibrium position monotonically decreases with the increase in particle size. The negative slope of these curves, i.e. the gradient of relative specific gravity per unit radial distance $\frac{d}{dr}\left(\frac{\sigma - p}{p}\right)$, is a measure of the separation efficiency. Fig. 5 shows the distribution of particle sizes as a function of equilibrium radial position, for different relative specific gravity, namely 0.75, 1, 1.25 and 1.5 respectively. These

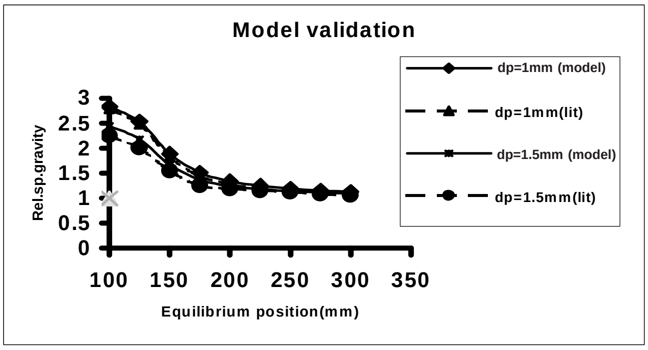

Fig. 4 : Predicted radial distribution of relative specific gravity and validation with published data[1].

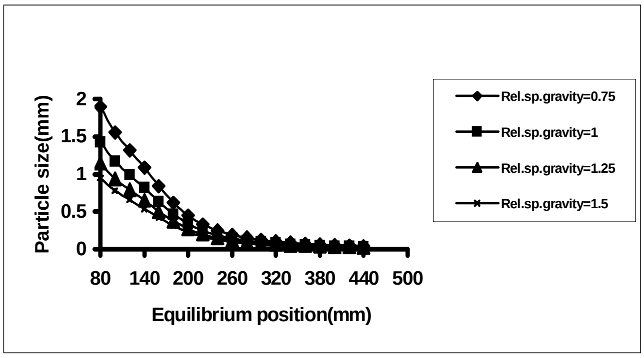

Fig. 5 : Effect of relative specific gravity on the radial distribution of particle size (feed mean flow depth = 5 mm).

show, for lower relative specific gravity values, gradient of particle size distribution along the equilibrium radial distance is higher. Figs. 6 and 7 show the particle relative specific gravity variation as a function of mean flow depth at various equilibrium positions (100 mm, 200 mm, 300 mm and 400 mm) for different particle sizes, namely, 1 mm and 2 mm respectively. It is observed that for all radial equilibrium positions, the relative specific gravity decreases with increase in particle size as a function of mean flow depth. However, the relative specific gravity increases almost linearly with mean flow depth for any given particle size.

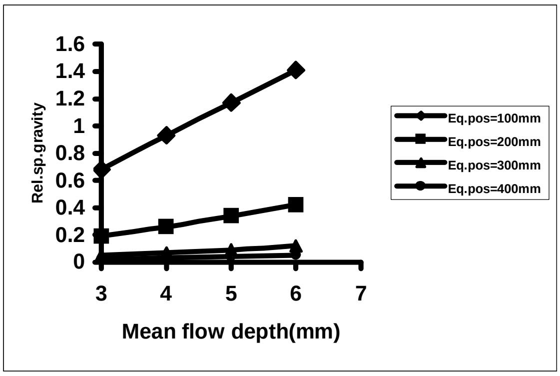

Fig. 6 : Sensitivity of relative specific gravity with mean flow depth at various equilibrium positions (particle size =1 mm).

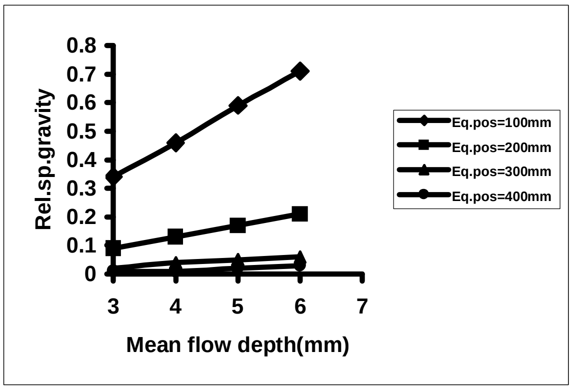

Fig. 7 : Sensitivity of relative specific gravity with mean flow depth at various equilibrium positions (particle size = 2 mm).

CONCLUSION

The spiral model presented in this paper incorporates three components of equilibrium force balance formalism. They are; spiral geometry, flow analysis and balances of forces acting on the particle including important Bagnold effect. The present model provides an exploratory foray for analysis of spiral separation characteristics and has considerable scope for improvement and refinement. The Bagnold effect is critical in the force balance analysis, when the particle concentration in the field is more than 10 % by volume. The semi-empirical approach for fluid dynamic analysis is justified by the need for developing a working equilibrium force model for an operating spiral with a complex spectrum of flow regimes. Attempts will be made in future to tune the adjustable parameter with the plant data for direct implementation of the model. The results are encouraging to infer that the model presented here is capable of predicting separation characteristics of coal-washing spiral on the basis of equilibrium distribution of relative specific gravity.

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